



Express Mail No. EV442675628 US
Application No. 09/665,179
Title: Method & Apparatus for Determining
Colimits of Hereditary Diagrams
Inventors: Dusko Pavlovic, et al.
Atty. Docket No.: 11128-04483

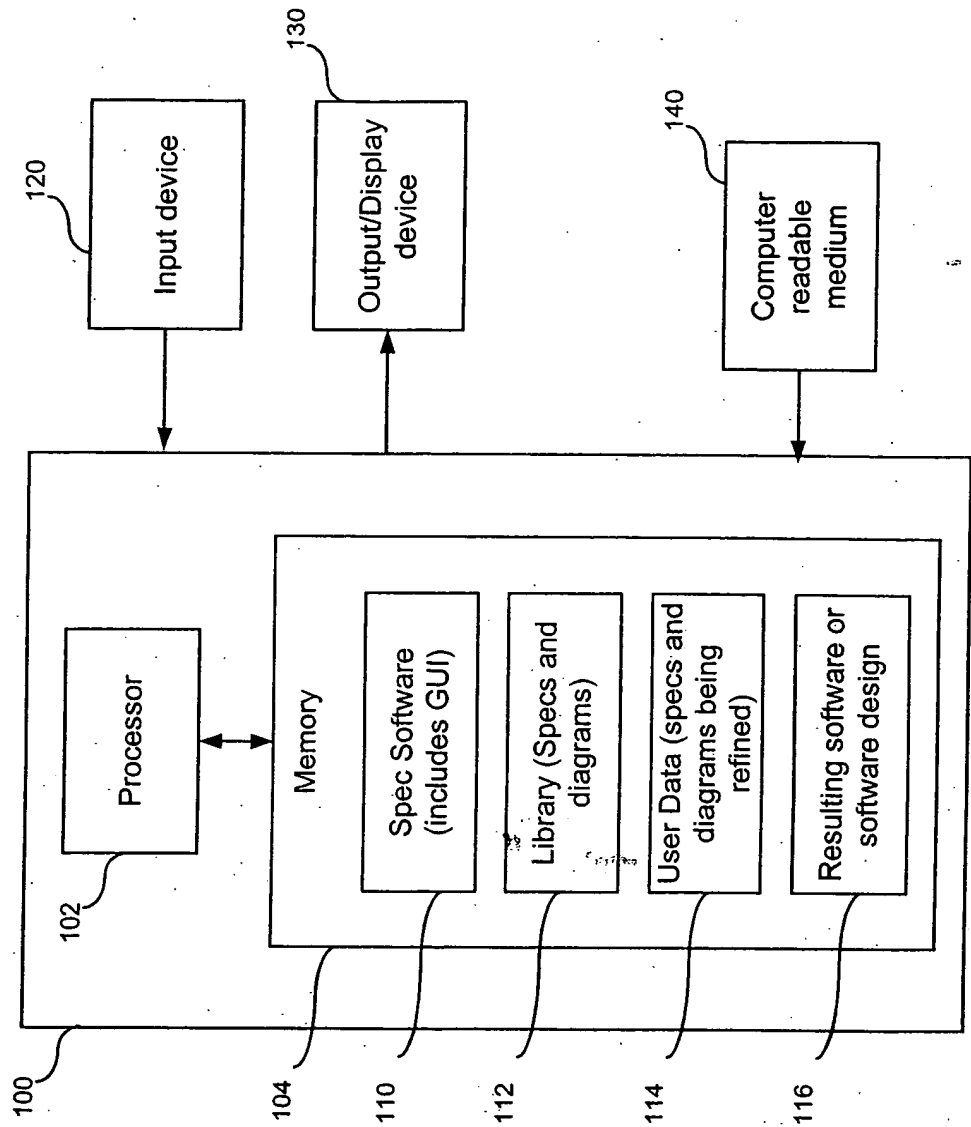


Fig. 1



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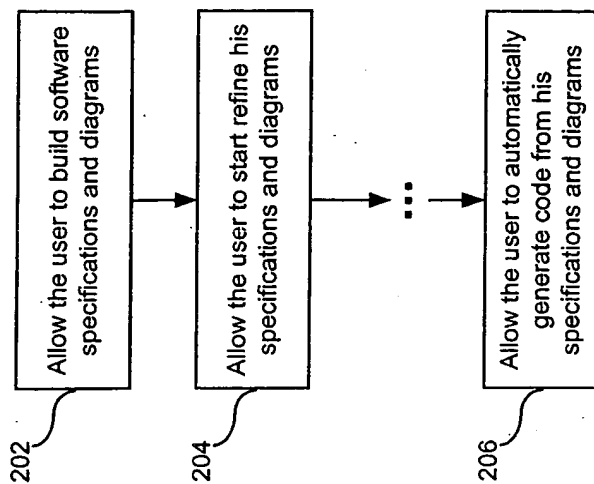
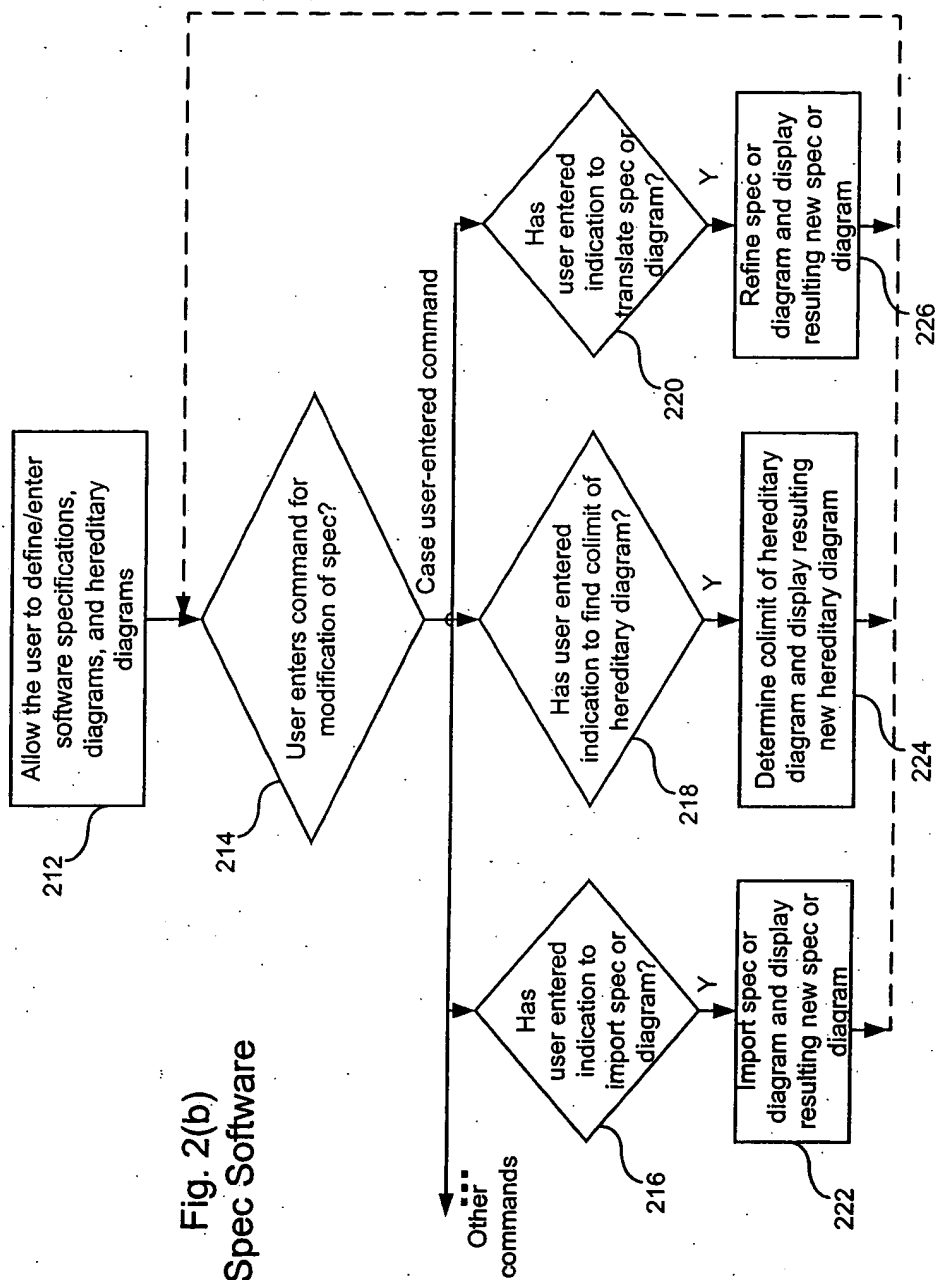
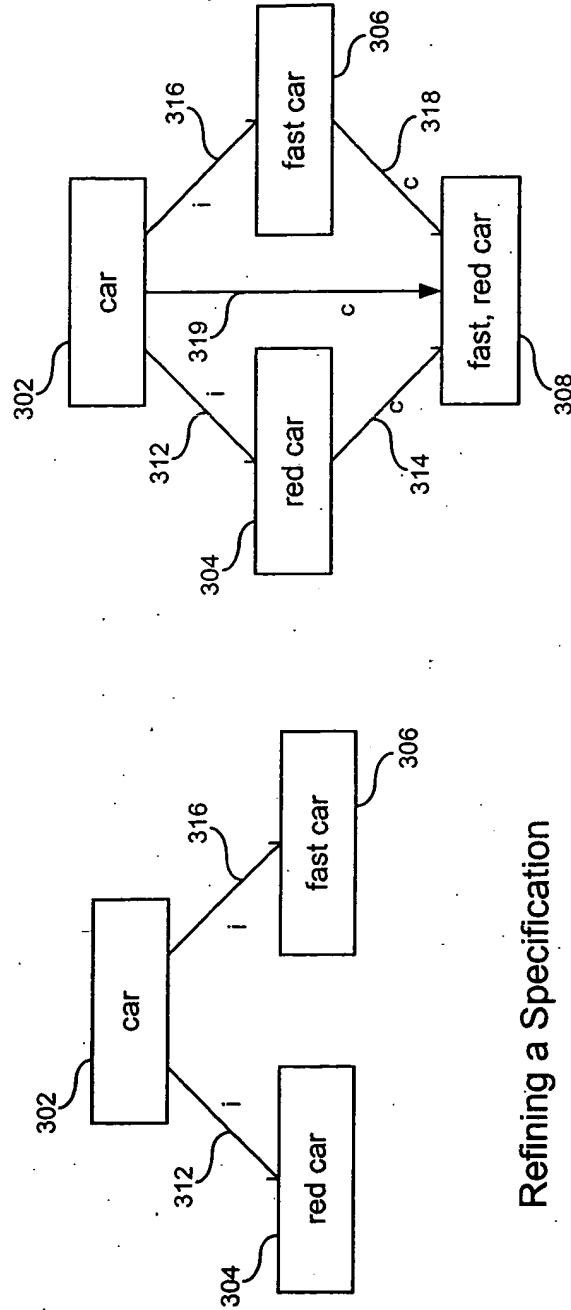


Fig. 2(a)
Spec Software





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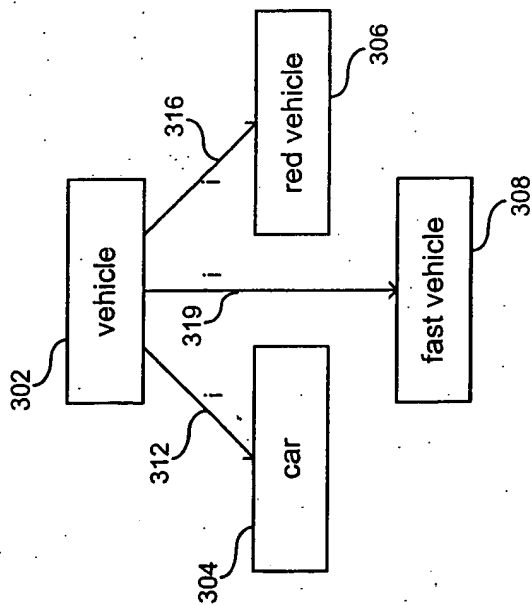


Refining a Specification
Fig. 3(a)

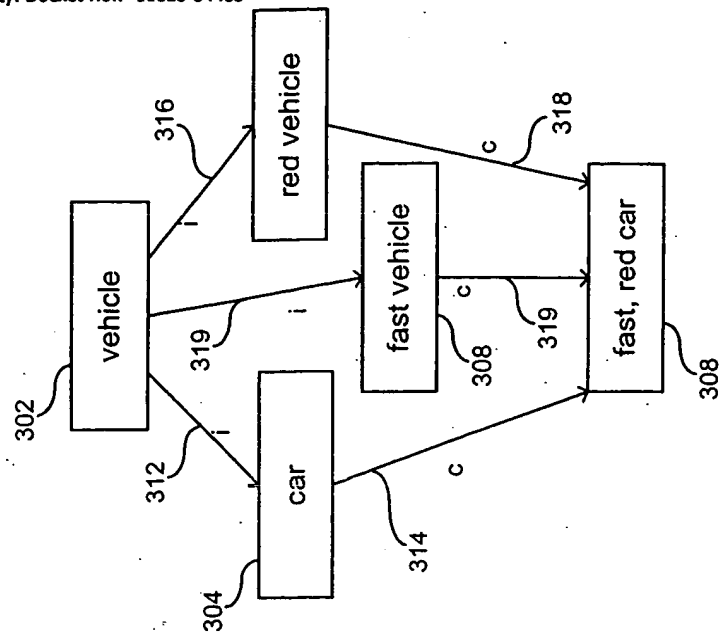
Example of Using a Colimit to
Combine Refined Specifications
Fig. 3(b)



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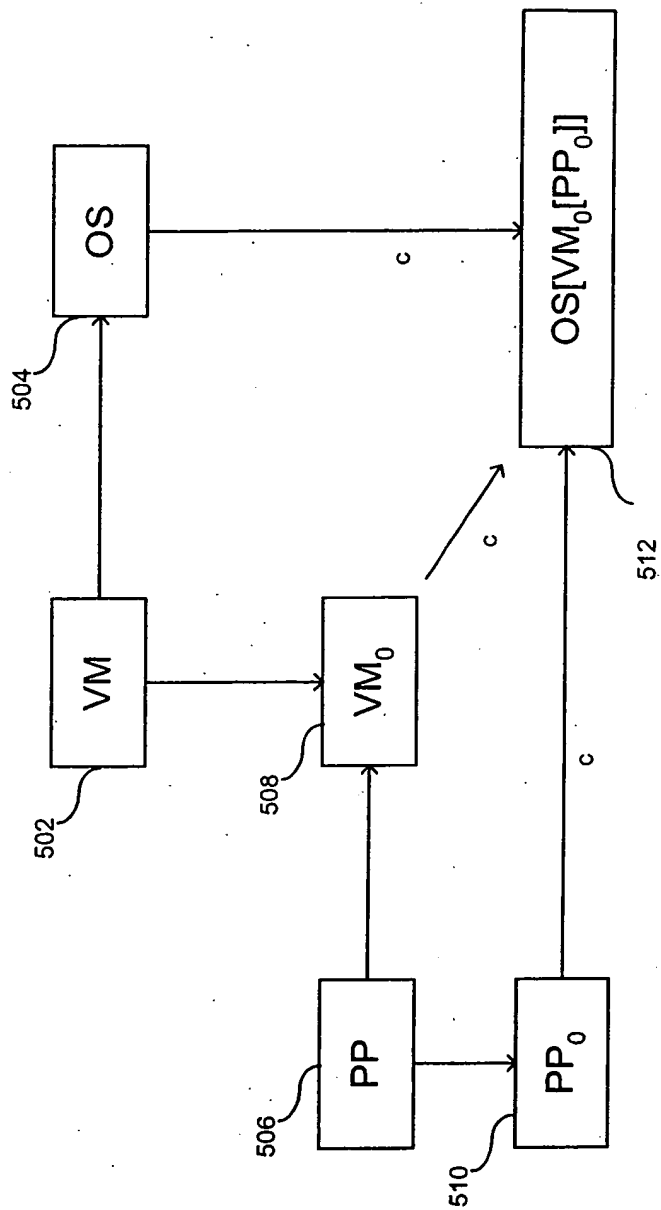
Refining a Specification
Fig. 4(a)



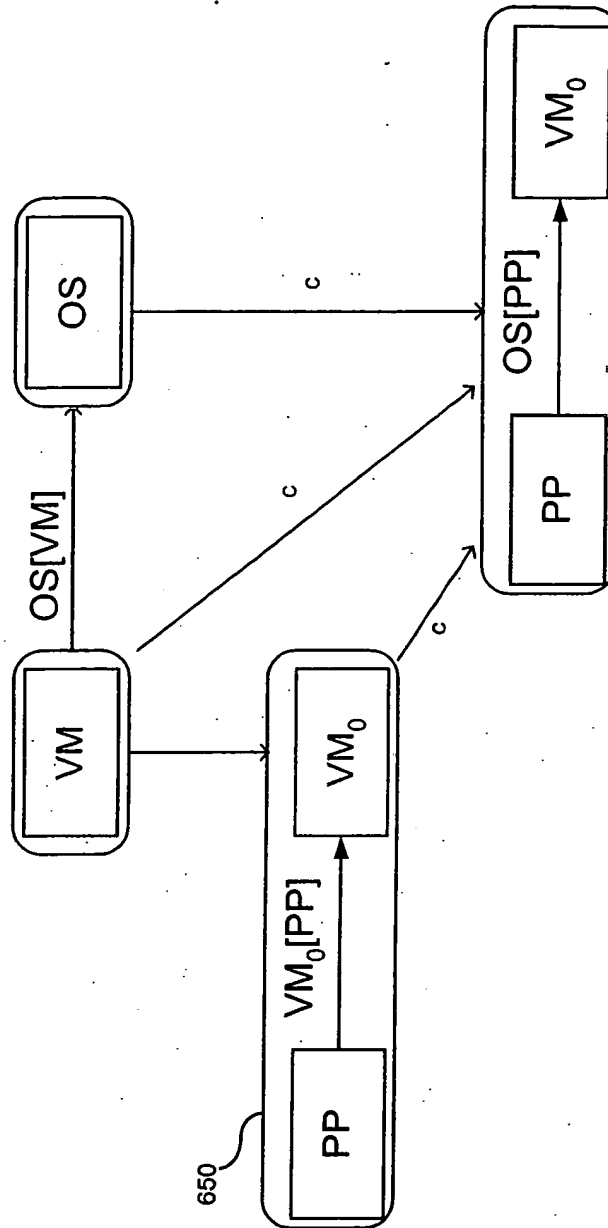
Example of Using a Colimit to
Combine Refined Specifications
Fig. 4(b)



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Example Colimit of Specifications
Fig. 5



Example Colimit of Diagrams
Fig. 6

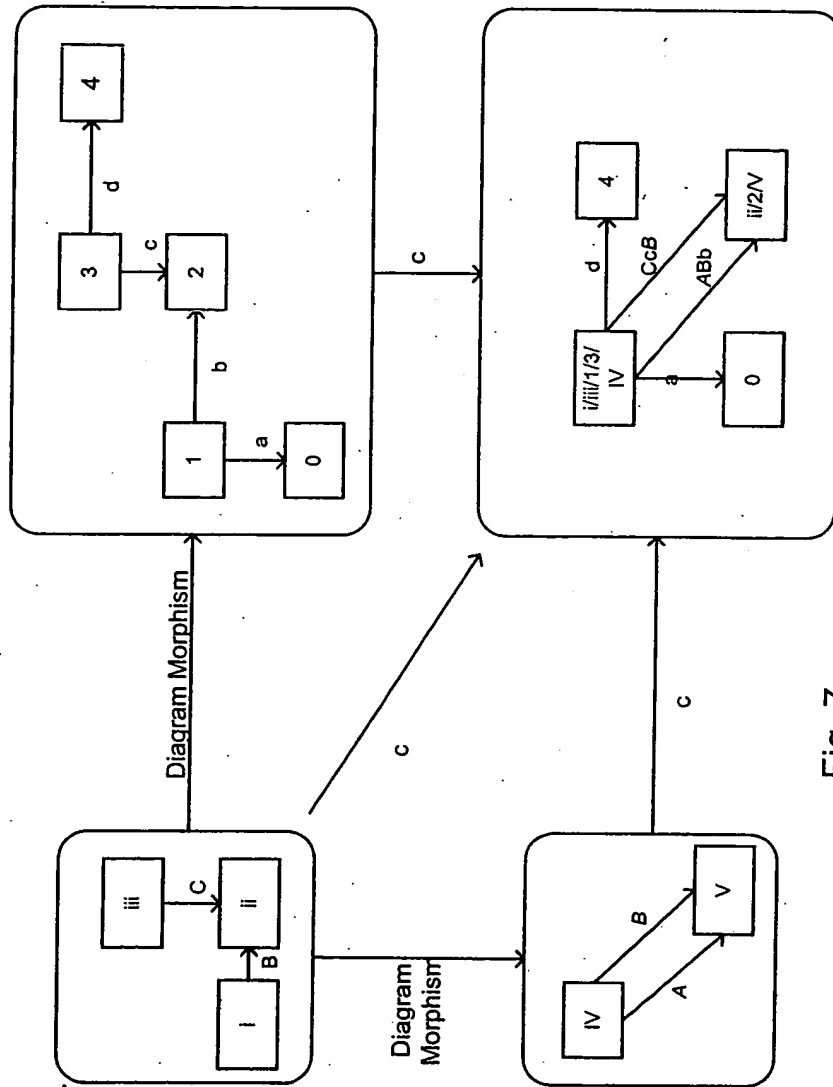
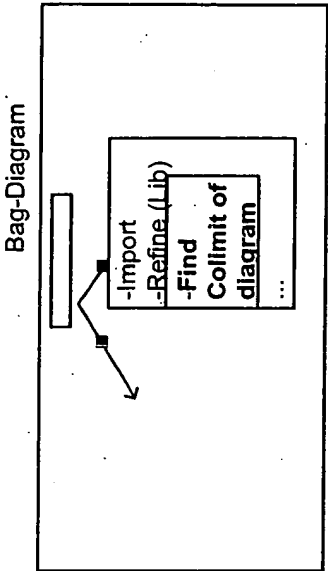


Fig. 7

Example of Taking the Colimit of Hereditary Diagrams

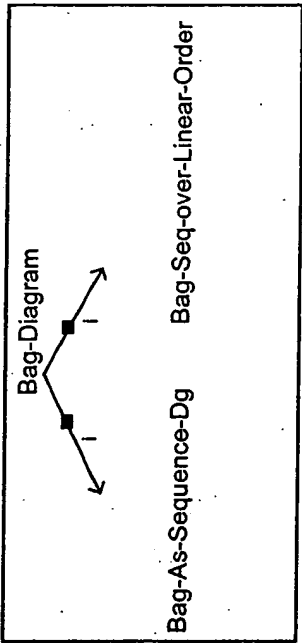


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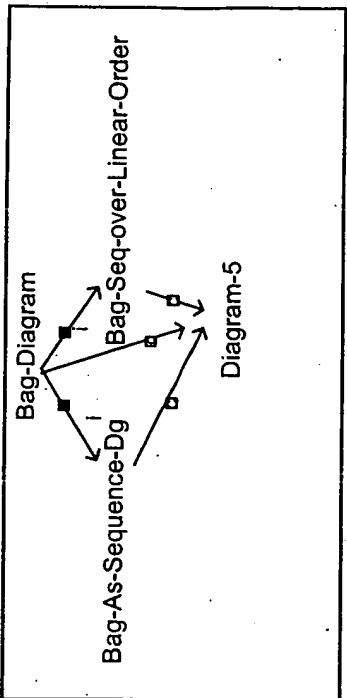
Example user interface showing a hereditary diagram (interface for user to indicate "find colimit" operation)

Fig. 8(b)



Example user interface showing a hereditary diagram

Fig. 8(a)

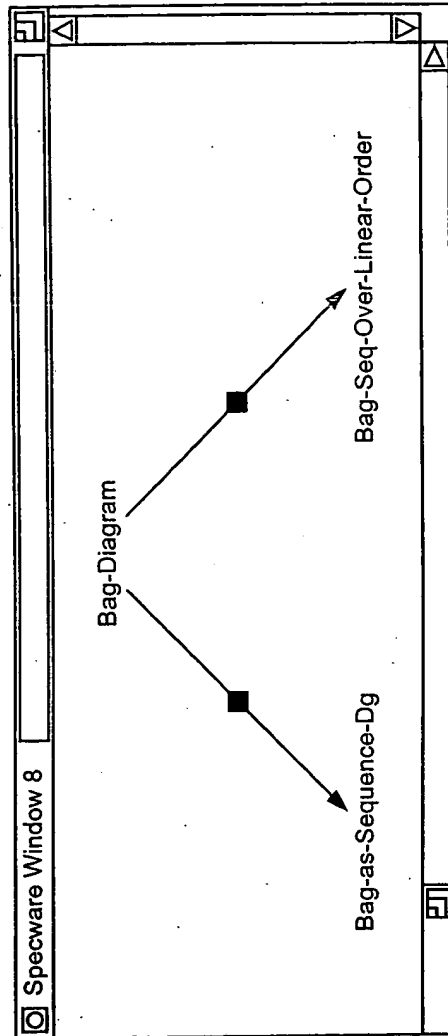


Example user interface showing a hereditary diagram after the user indicates a "find colimit" operation for the hereditary diagram and the colimit operation is performed

Fig. 8(c)



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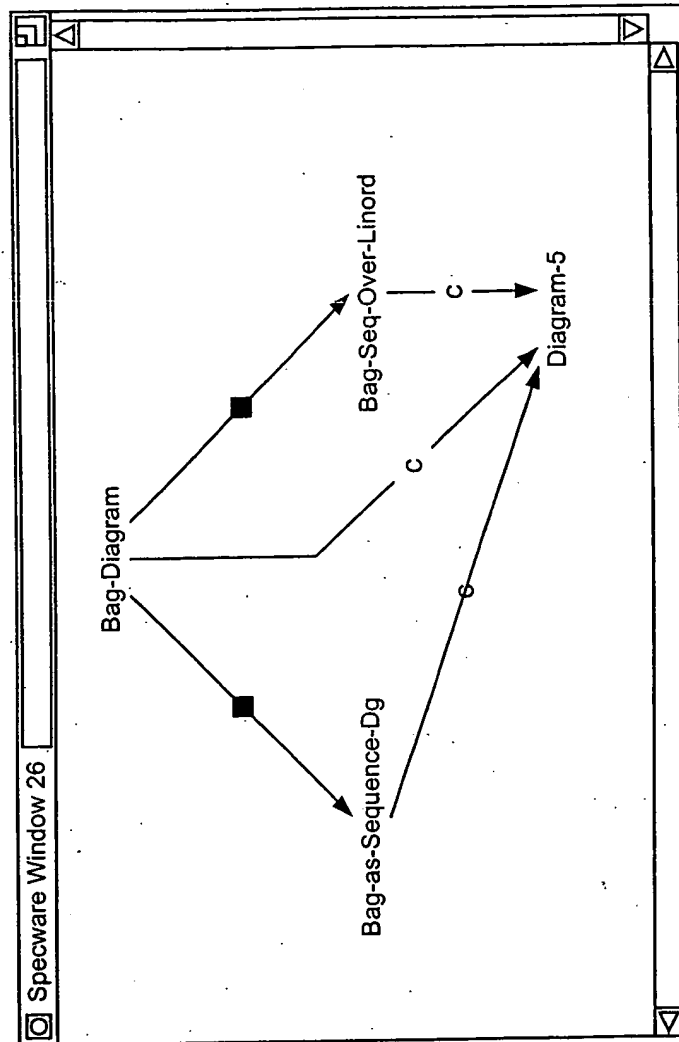


Hereditary diagram

Fig. 9(a)



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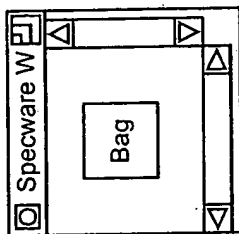


Hereditary diagram, including colimit

Fig. 9(b)



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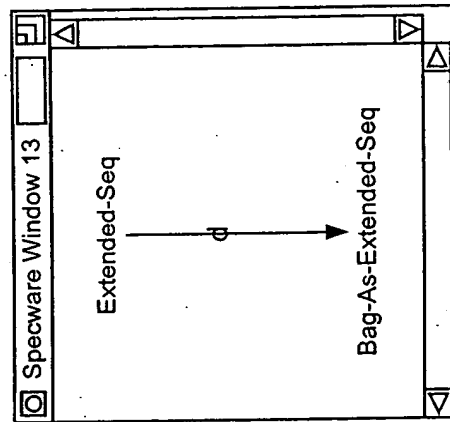


Bag diagram
(obtained by expanding node
Bag-Diagram
in Hereditary diagram)

Fig. 9(c)

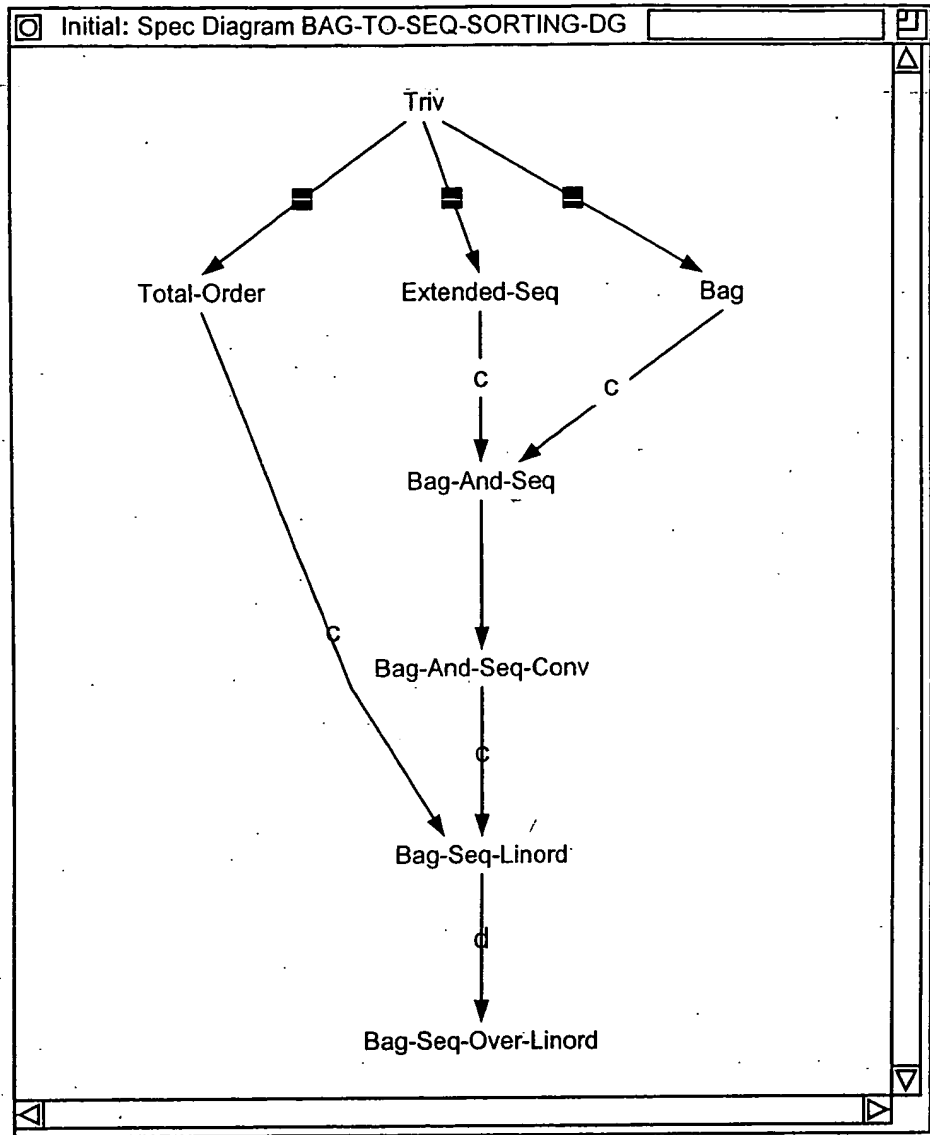


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Bag-as-Sequence diagram
(obtained by expanding node
Bag-as-Sequence-diagram
in Hereditary diagram)

Fig. 9(d)

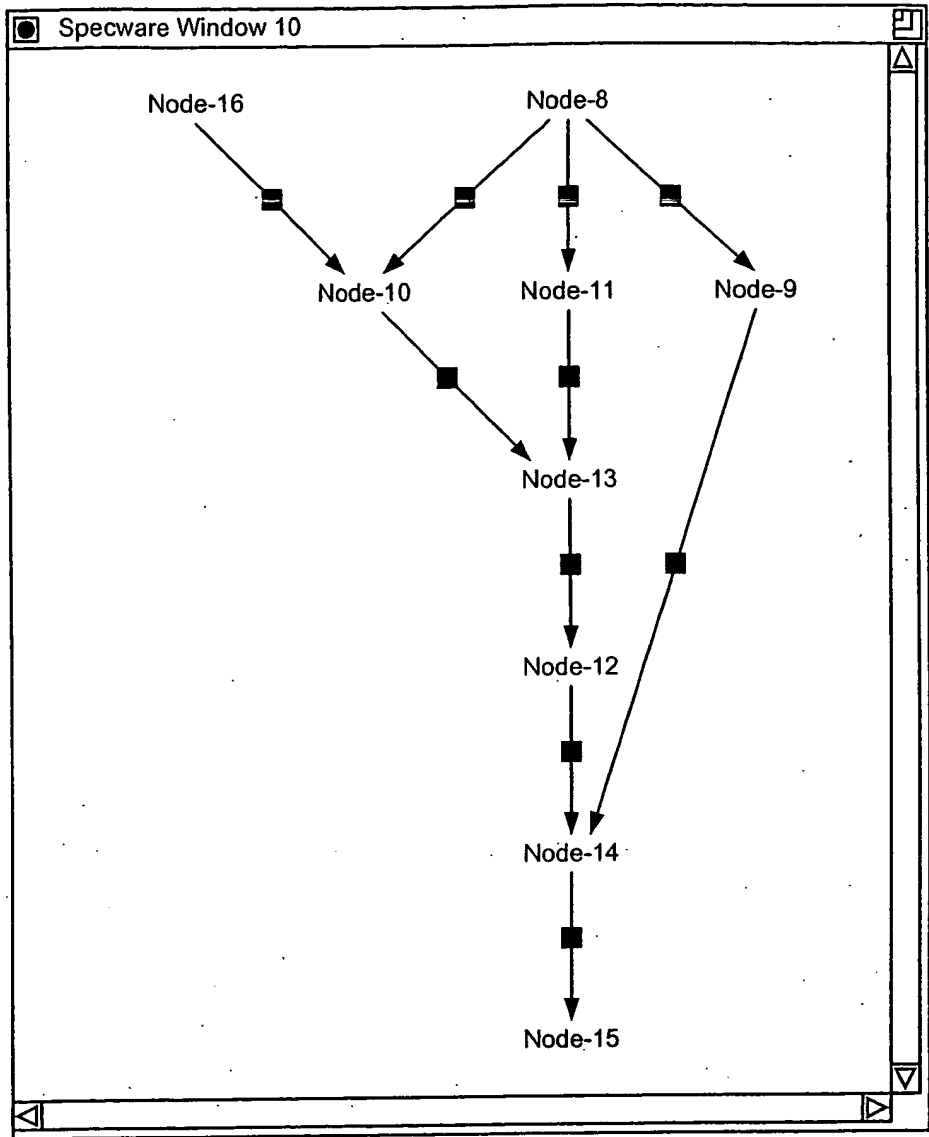


Bag-Seq-over-Linear-Order diagram
(obtained by expanding node
Bag-Seq-over-Linear-Order-
diagram
in Hereditary diagram)

Fig. 9(e)



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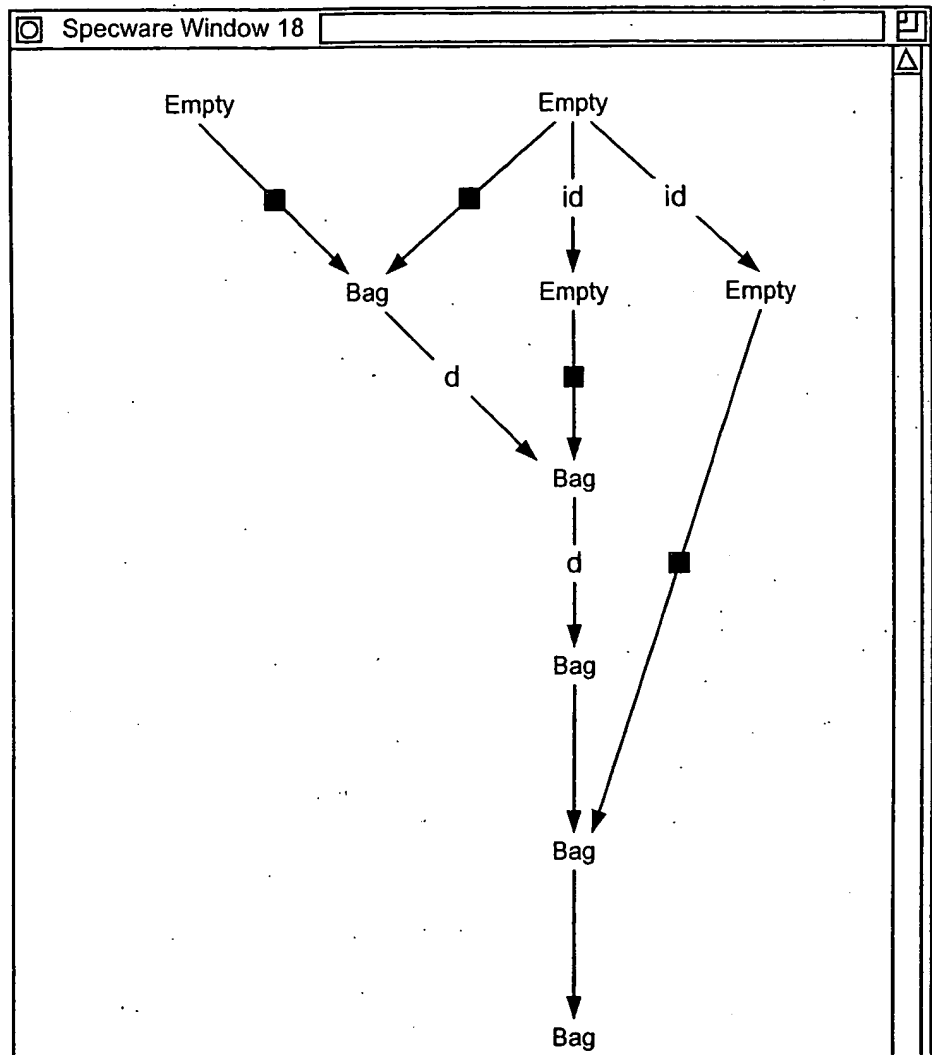


Shape of colimit

Fig. 9(f)

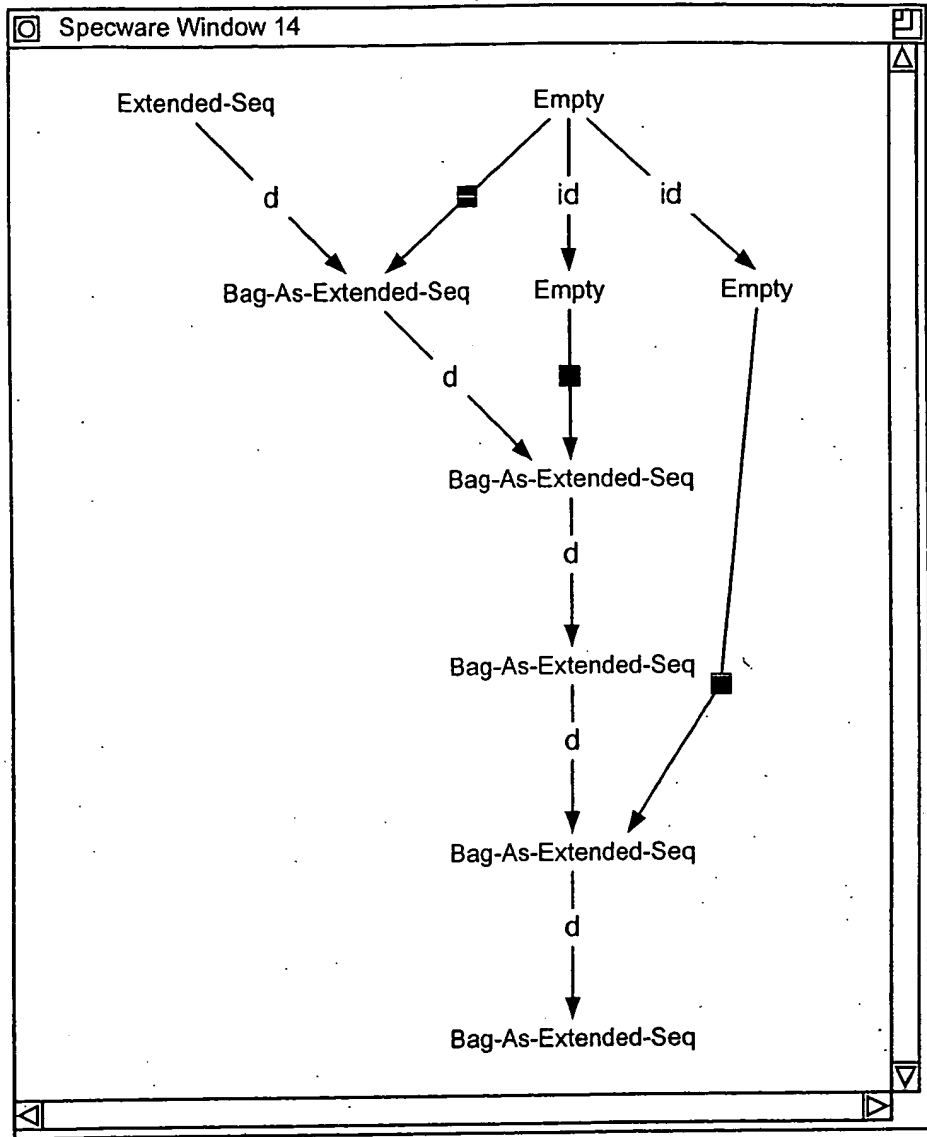


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Extended Bag diagram

Fig. 9(g)

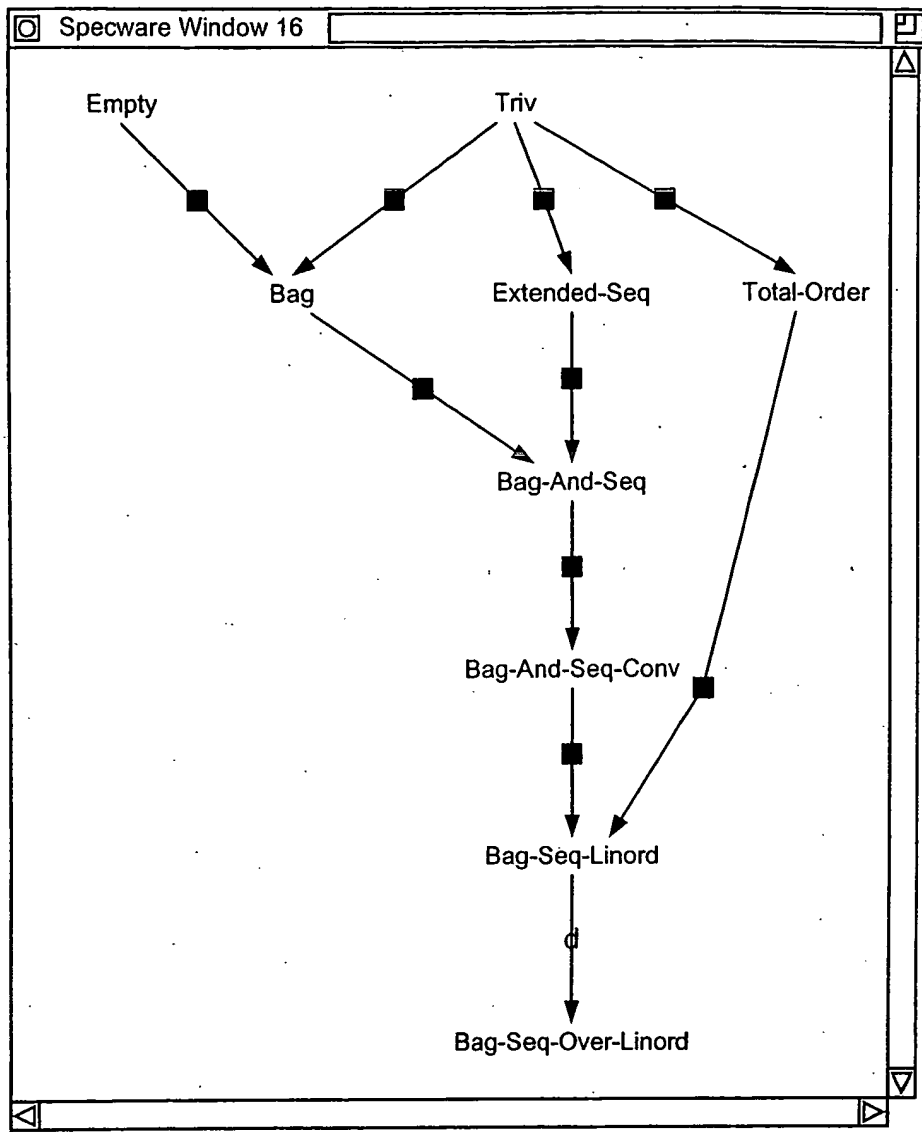


Extended Bag-as-Sequence diagram

Fig. 9(h)

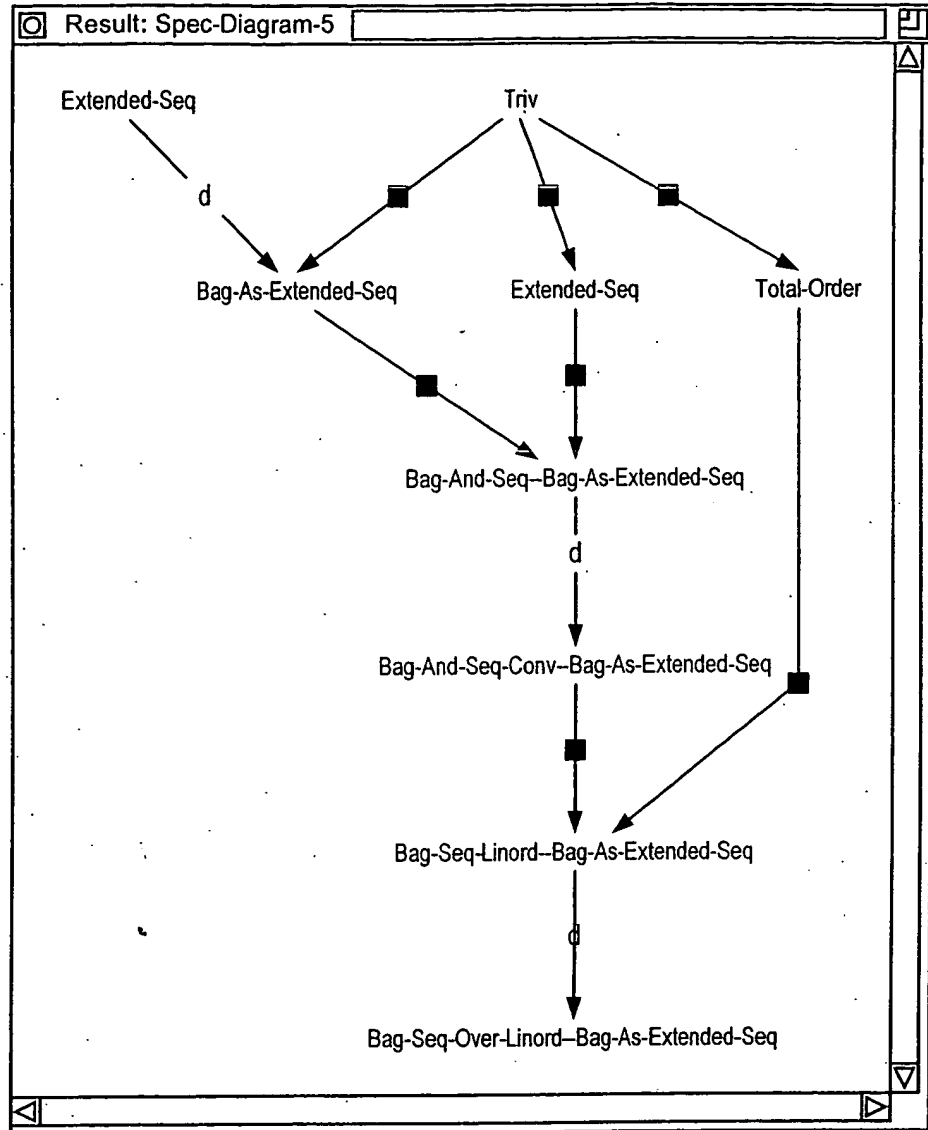


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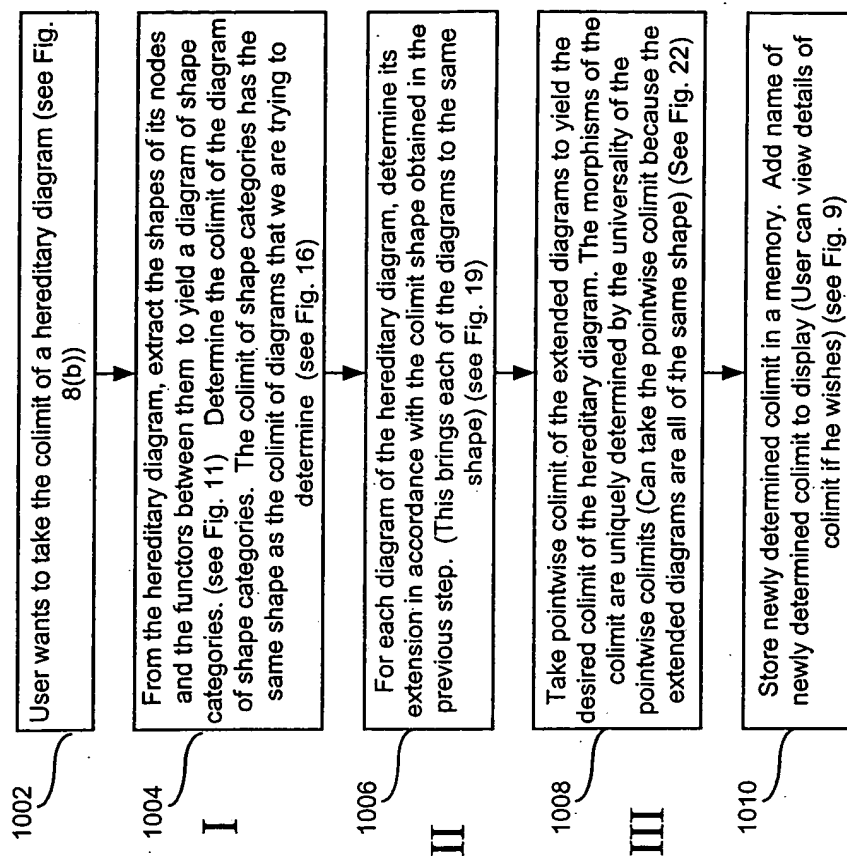
Extended Bag-Seq-over-Linear-Order diagram

Fig. 9(i)



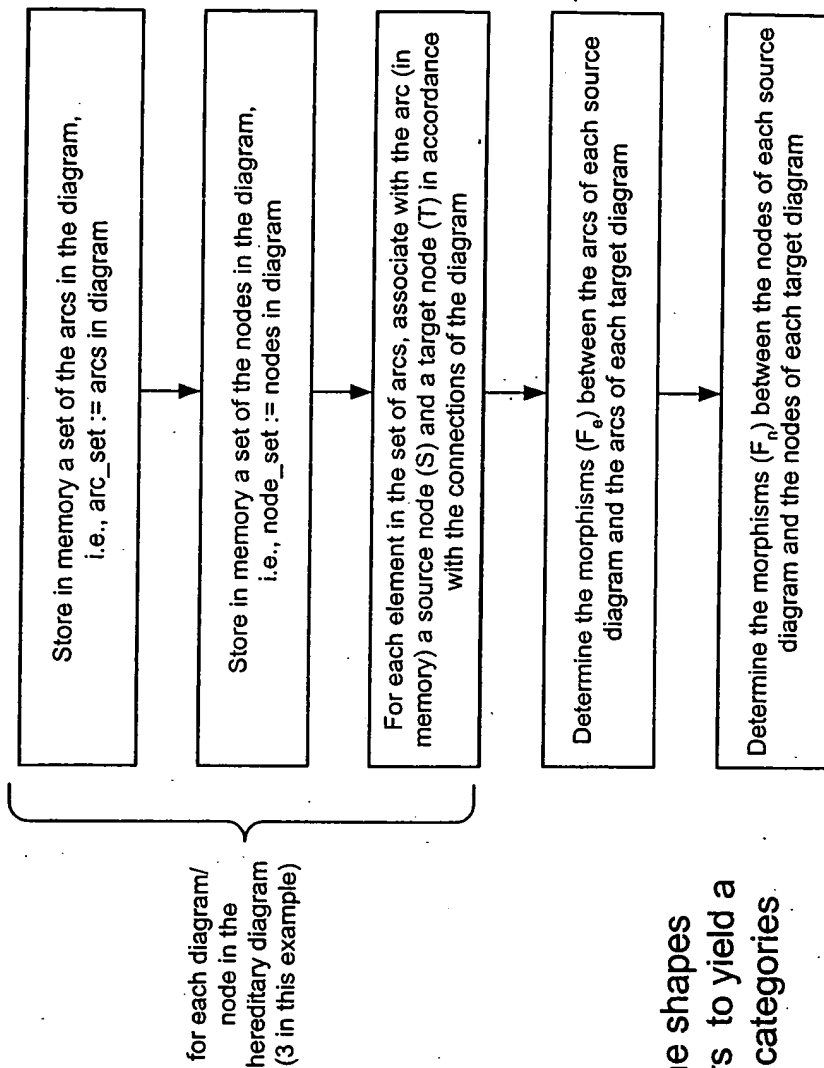
Colimit of Hereditary diagrams
(final result)

Fig. 9(j)



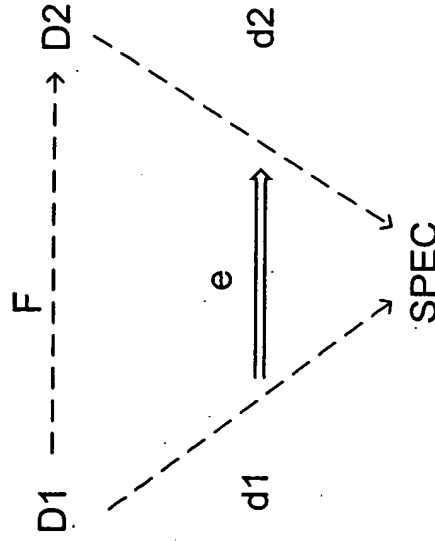
Finding a Colimit of
Hereditary Diagrams

Fig. 10



PART I: Extract the shapes
and shape functors to yield a
diagram of shape categories

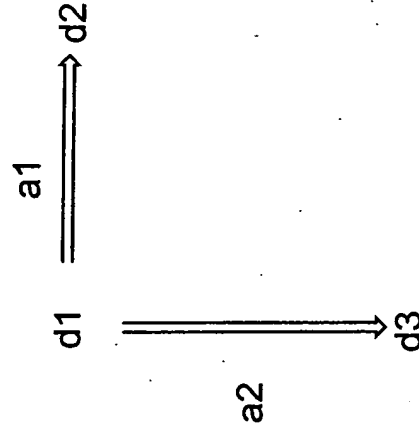
Fig. 11



A Shape Morphism

where $d1$ and $d2$ are diagrams,
 F is a shape functor,
 e is a natural transformation from $d1$ to
 $(d2$ composed with $F)$
 $D1$ and $D2$ are shape categories of
 diagrams, and $SPEC$ is the category

Spec
 Fig. 12(b)



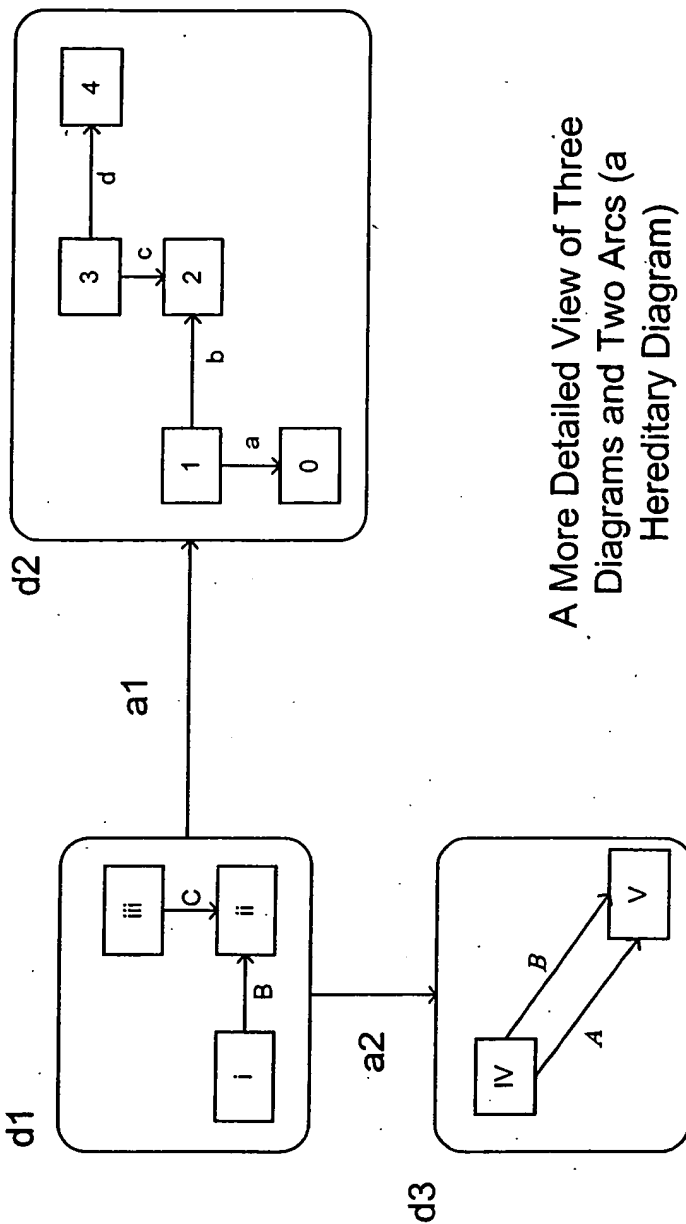
A Hereditary Diagram: Three
 Diagrams and Two Arcs.

Each arc $a1$ and $a2$ represents a
 shape morphism having 1) a
 shape functor (such as F) and 2)
 a natural shape transformation
 (such as $e: d1 \rightarrow d2$)

Fig. 12(a)



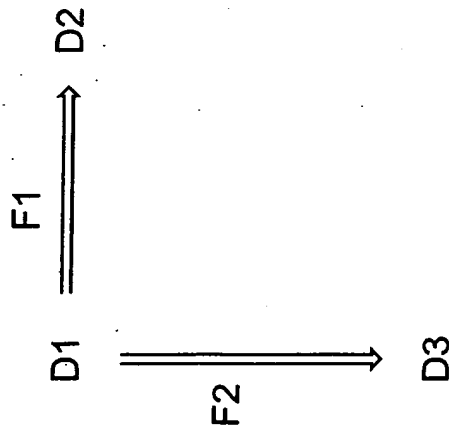
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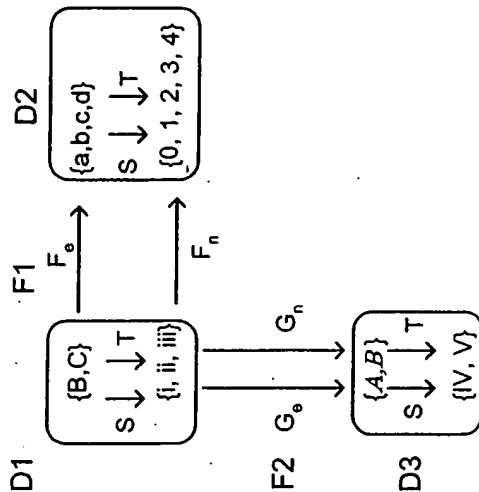
A More Detailed View of Three
Diagrams and Two Arcs (a
Hereditary Diagram)
Fig. 13



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Extract the Shapes and
 Shape Functors
 (D1 is shape of diagram
 d1, F1 is shape functor)
 Fig. 14



More Detailed View of Extracting the
 Shapes and Shape Functors
 (continued on Figs. 15(b)-15(d))
 Fig. 15(a)

Arcs: B -> A
 C -> B
 Nodes: i -> IV
 ii -> V
 iii -> IV

Mapping for F2
 Fig. 15(c)

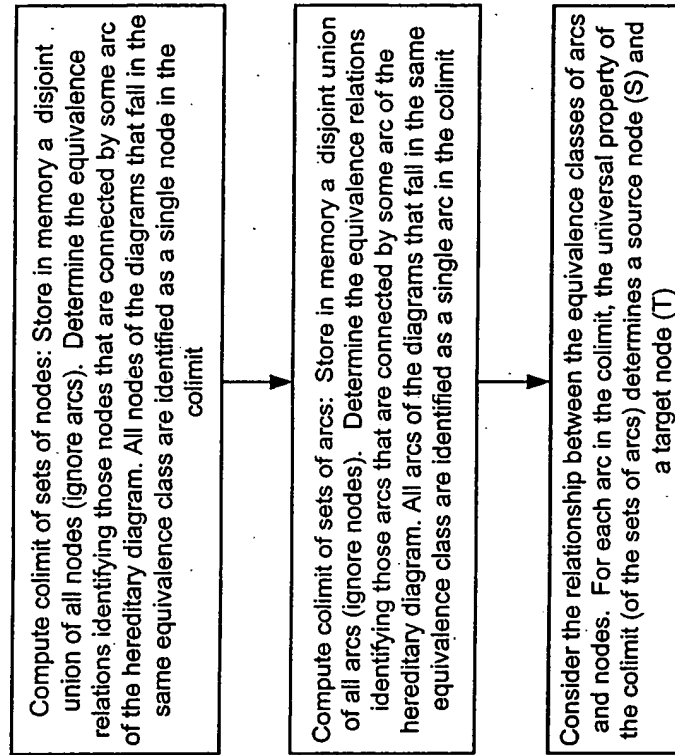
Arcs: B -> b
 C -> c
 Nodes: i -> 1
 ii -> 2
 iii -> 3

Mapping for F1
 Fig. 15(b)

Source (S) and
 Target (T)
 Functions for
 Hereditary
 Diagrams
 Fig. 15(d)

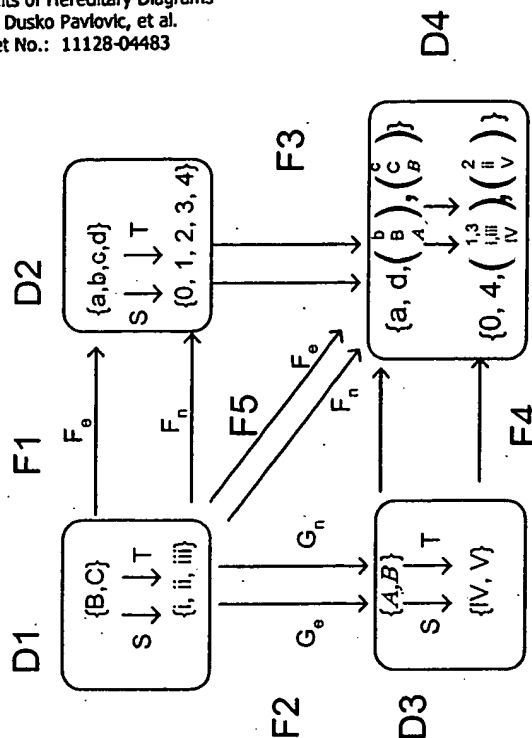
Arc	B	C	a	b	c	d	A	B
Source	i	iii	1	1	3	3	IV	IV
Target	ii	ii	0	2	2	4	V	V





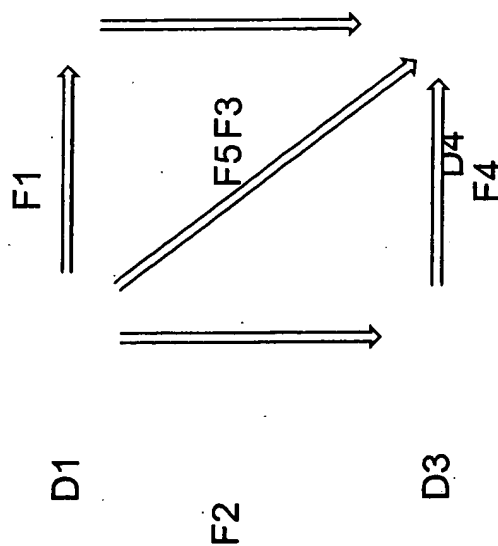
PART I: Determine the colimit of the diagram of shape categories.

Fig. 16

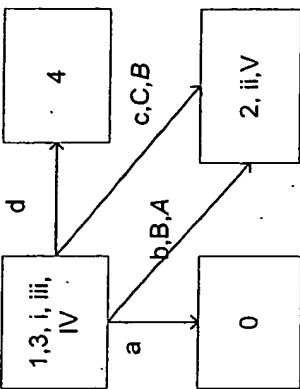


More Detailed View of Taking the Colimit
 (See Figs 18(b)-(f))

Fig. 18(a)



More Detailed View of
 Taking the Colimit
 Fig. 17



Source (S) and
Target (T)
Functions for
Shape Colimit D4
Fig. 18(b)

Arc	a	d	b	c
Source	1,3, i, iii, IV	1,3, i, iii, IV	A, A, 1,3, i, iii, IV	C, C, B, 1,3, i, iii, IV
Target	0	4	2, ii, V	2, ii, V

Arcs: a -> a
d -> d
b -> b, B, A
c -> c, C, B

Nodes:
0 -> 0
1 -> 1,3, i, iii, IV
2 -> 2, ii, V
3 -> 1,3, i, iii, IV
4 -> 4

Mapping for F3
Fig. 18(d)

The Colimit D4 of the Shape Diagrams
Fig. 18(c)

Arcs: A -> b, B, A
B -> c, C, B

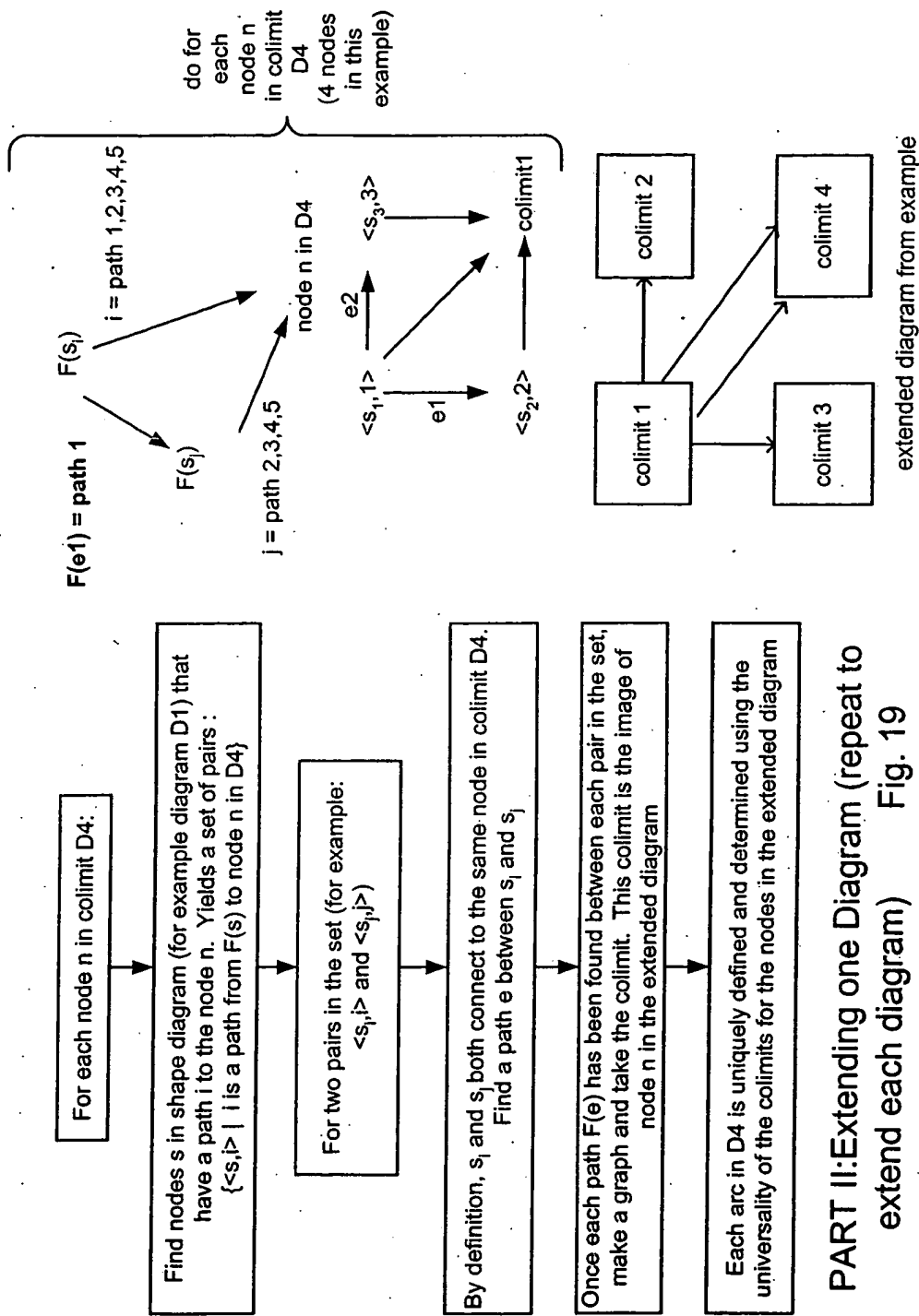
Nodes:
IV -> 1,3, i, iii, IV
V -> 2, ii, V

Mapping for F4
Fig. 18(e)

Arcs: B -> b, B, A
C -> c, C, B

Nodes:
i -> 1,3, i, iii, IV
ii -> 2, ii, V
iii -> 1,3, i, iii, IV

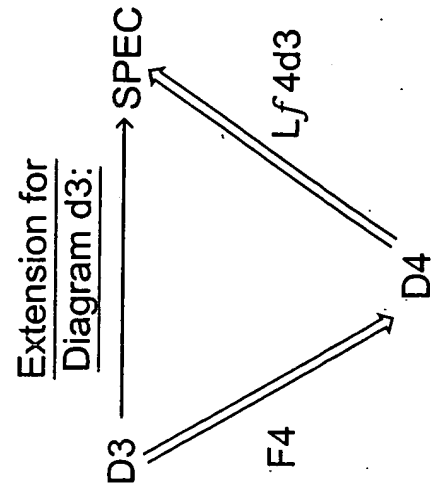
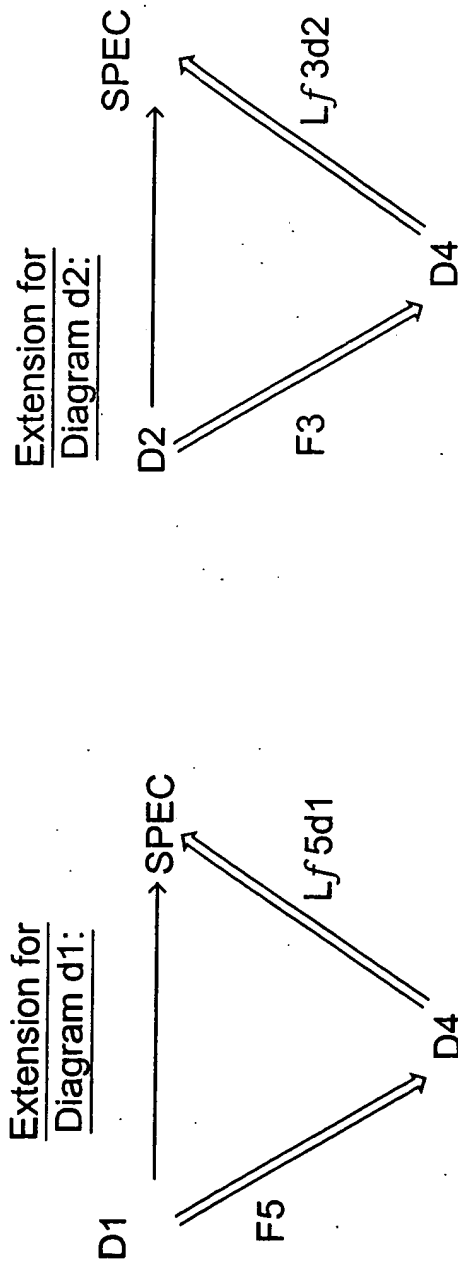
Mapping for F5
Fig. 18(f)



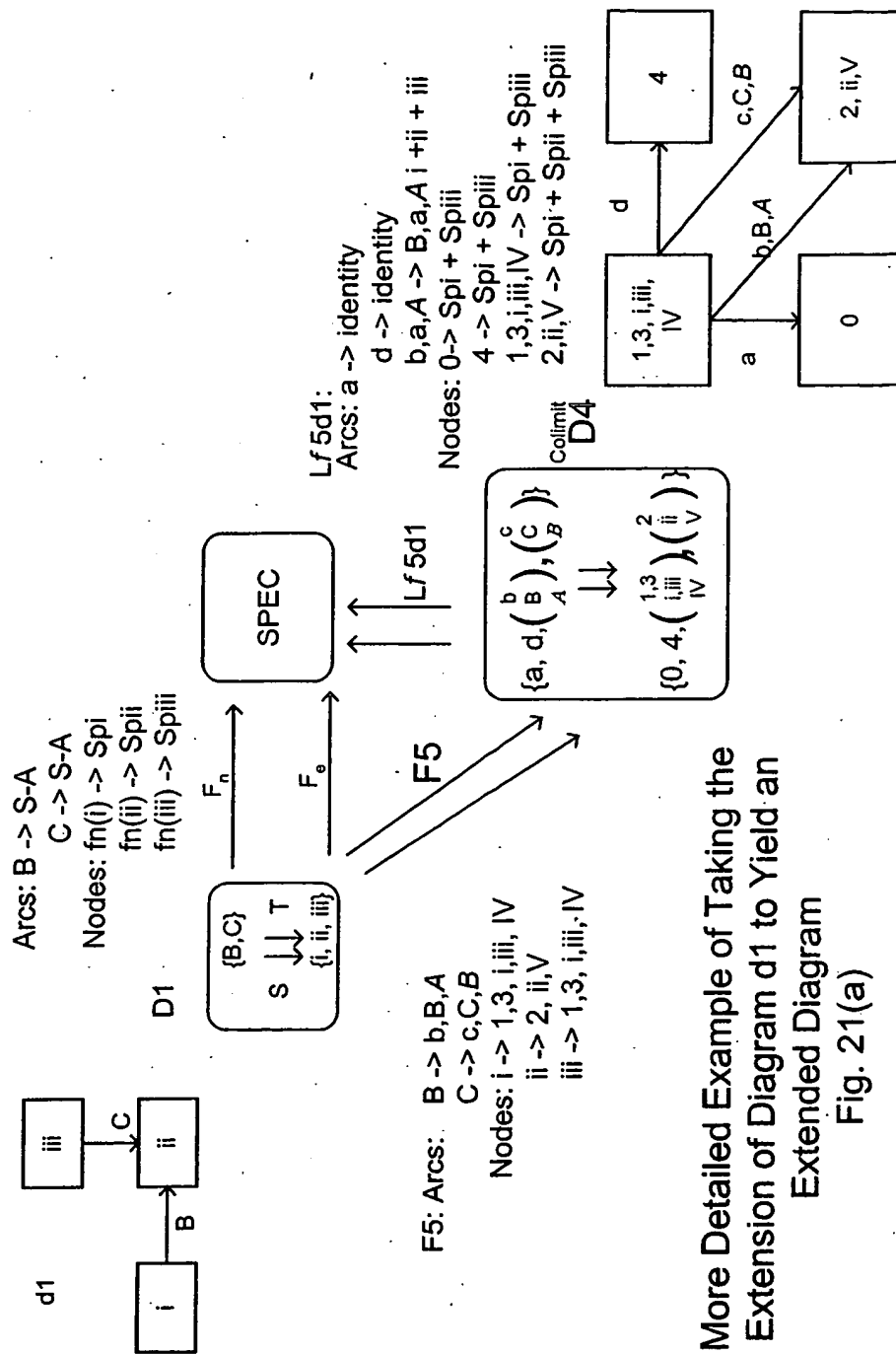
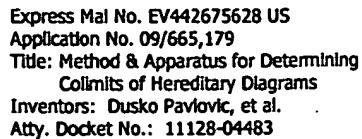
PART II: Extending one Diagram (repeat to extend each diagram) Fig. 19



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Example of Taking the
Extension of Each Node of the
Hereditary Diagram
Fig 20



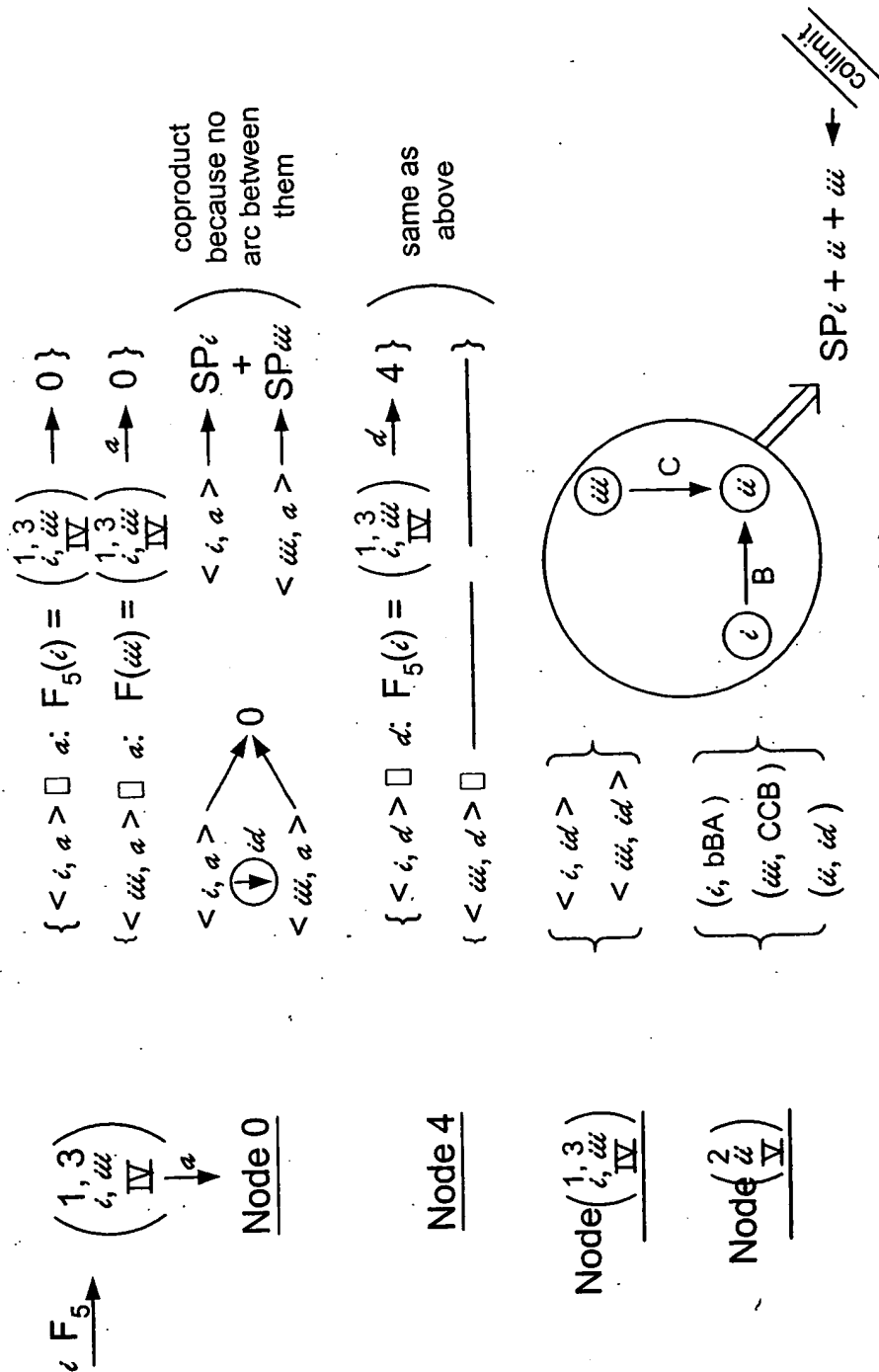
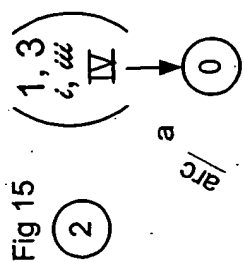
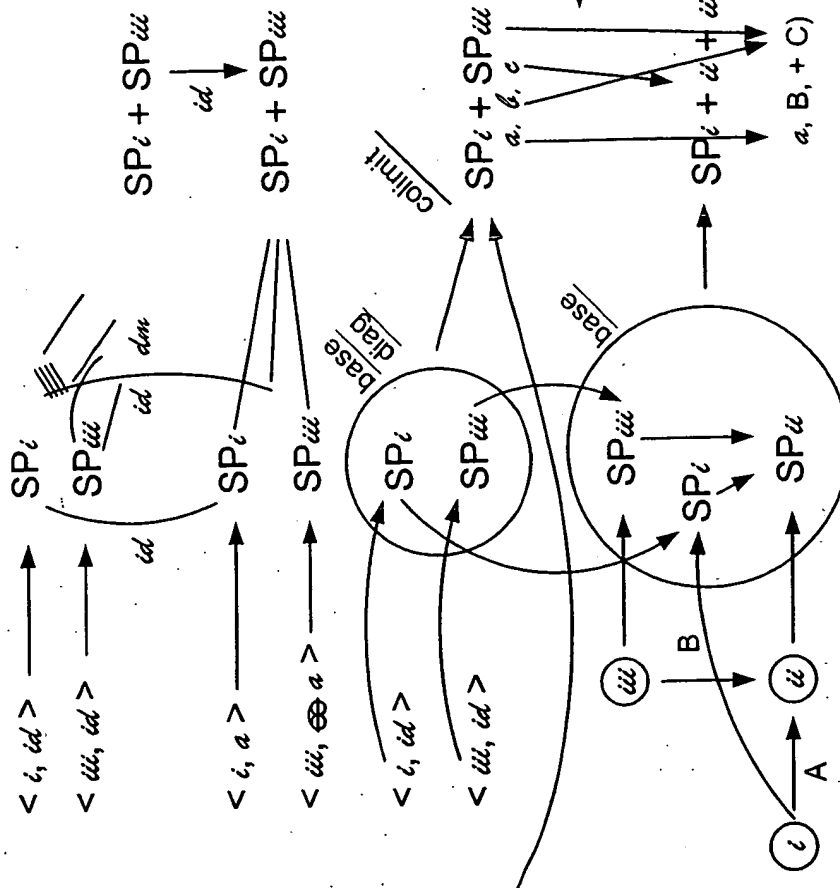


Fig. 21(b)

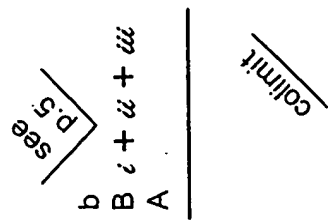
Example of Diagram Extension.



for arcs
in D4



Example of Diagram Extension (cont) Fig. 21(c)



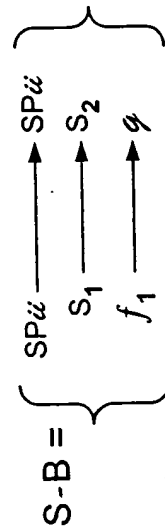
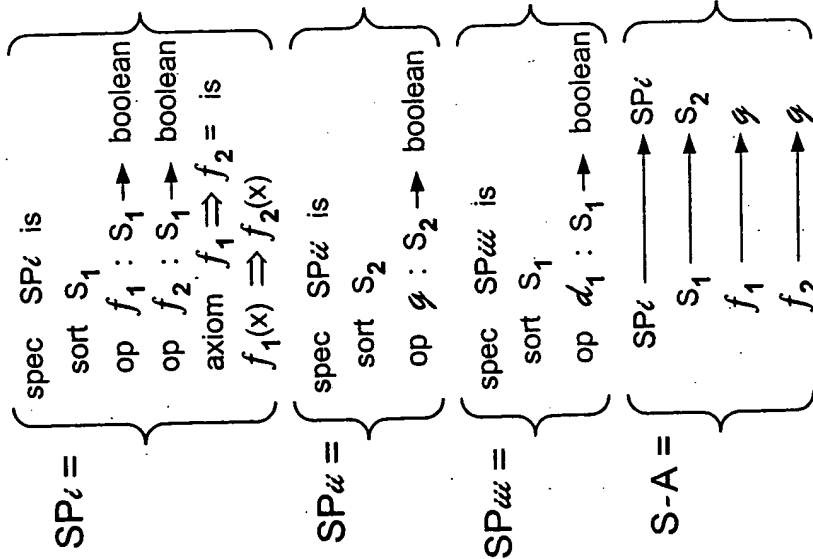
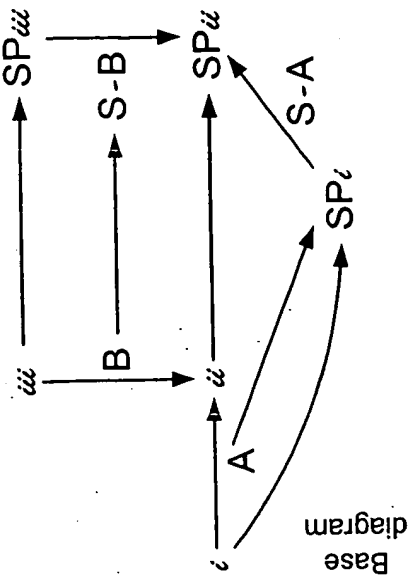
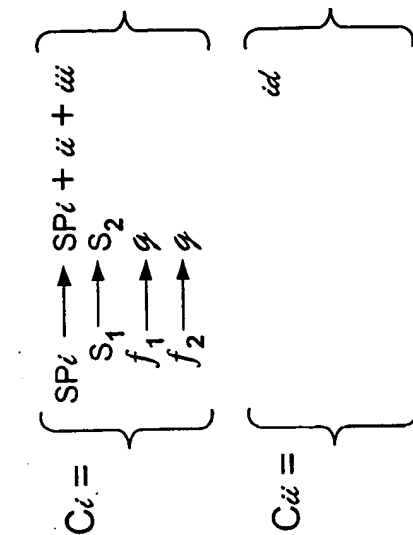
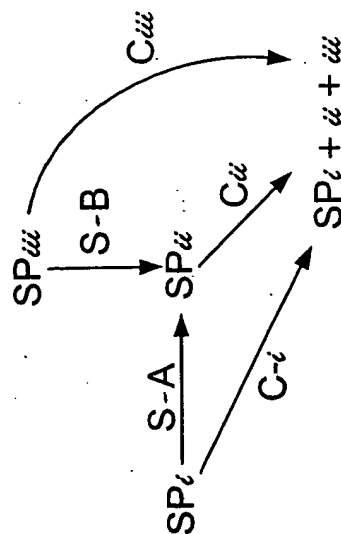


Fig. 21(d)

Example of Diagram Extension (cont)



$$C_{ii} = S-B$$



$$\text{colimit } \text{SP}_i + u + u = \left\{ \begin{array}{l} \text{spec } \text{SP}_i + u + u \text{ is} \\ \text{sort } \text{SP}_i, S_1, S_2 \\ \text{sort } \text{SP}_i, S_1 \\ \text{sort } \text{SP}_i, S_2 \\ \text{op } q : S_2 \rightarrow \text{boolean} \\ \text{axiom } q \cdot q \text{ is} \\ q(x) \Rightarrow q(x) \end{array} \right\}$$

Example of Diagram Extension (cont) **Fig. 21(e)**



$$SP_i + SP_{iii} = \left\{ \begin{array}{l} \text{sort } SP_i \cdot S_1 \\ \text{sort } SP_{iii} \cdot S_1 \\ \text{op } SP_i \cdot f_1 : SP_i \cdot S_1 \rightarrow \text{boolean} \\ \text{op } SP_{iii} \cdot f_1 : SP_{iii} \cdot S_1 \rightarrow \text{boolean} \\ \text{op } SP_i \cdot f_2 : SP_i \cdot S_1 \rightarrow \text{boolean} \end{array} \right\}$$

↓
 b
 B $i + ii + iii$
 A

$$SP_i + ii + iii = \left\{ \begin{array}{l} \text{See Fig. 21(e)} \end{array} \right\}$$

$$\begin{array}{l} b \\ B \\ A \end{array} \quad i + ii + iii = \left\{ \begin{array}{l} SP_i \cdot S_1 \longrightarrow S_2 \\ SP_{iii} \cdot S_1 \longrightarrow S_2 \\ SP_i \cdot f_1 \longrightarrow q \\ SP_{iii} \cdot f_1 \longrightarrow q \\ SP_{iii} \cdot f_2 \longrightarrow q \end{array} \right\}$$

Example of Diagram Extension (cont)

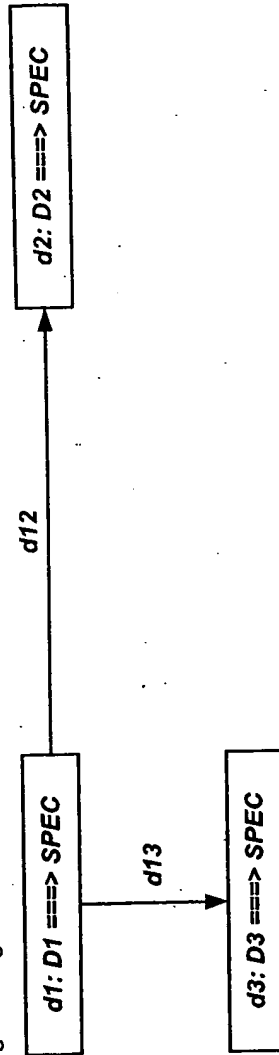
Fig. 21(f)



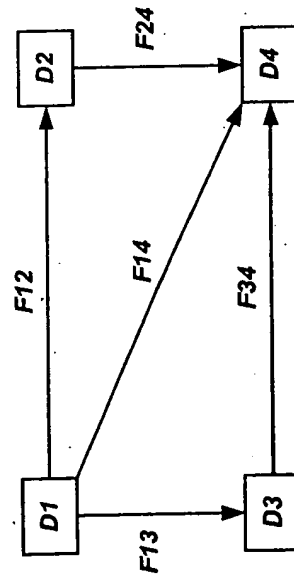
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After finishing the extension for each diagrams, let us use the following example:

Original diagrams:

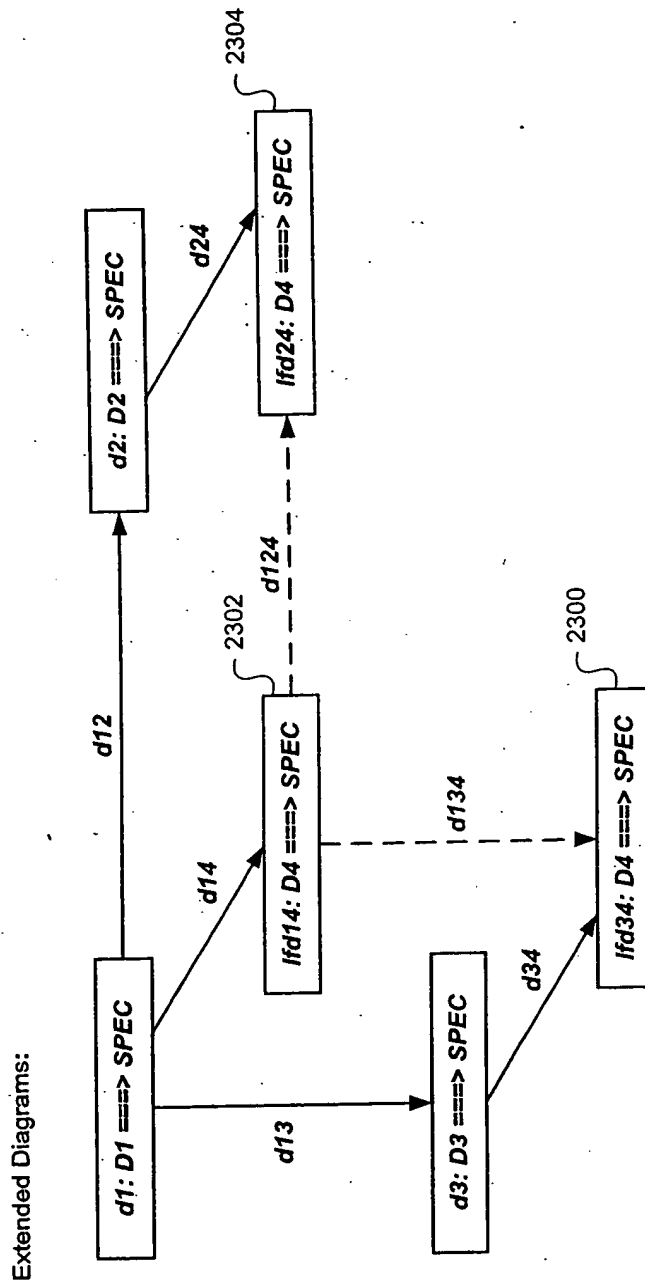


Its underlying shape categories, shape functors and the colimit are:



Part III

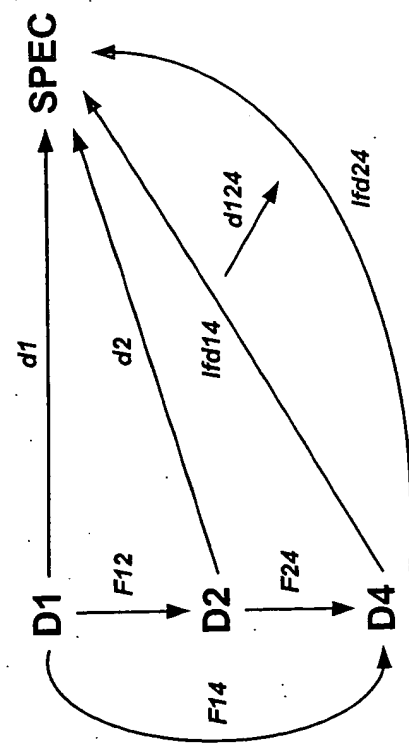
Fig. 22



The last algorithm step we are missing for constructing the diagram colimits is the diagram morphisms between extended diagrams. For example, the diagram morphism d_{124} and d_{134} (dotted lined arrows in above figure) are the ones needed.

Suppose lfd_{14} and lfd_{24} are two extensions of d_1 and d_2 , given the colimit of the shape categories as D_4 . We would have the following picture.

Fig. 23



A morphism between $lfd14$ and $lfd24$ is a natural transformation which maps each node of $D4$ to an arrow in $SPEC$. We ?? ???
the universal construction of witness arrows.

For any node ni in $D4$, we have $F14(ni) = F12 \circ F24(ni)$. Let $Sp1ni$ and $Sp2ni$ two shape categories used for constructing a mapping for ni in its extension of $d1$ and $d2$, respectively, then we can have a shape function between $Sp1ni$ and $Sp2ni$ (inclusion, basically). That induces a diagram morphism between the base diagrams for the target of ni in $lfd14$ and $lfd24$, respectively. By imposing that diagram morphism and cocone morphism, we can get an unique arrow between $lfd14(ni)$ and $lfd24(ni)$. Repeating this process we construct a natural transformation between $lfd14$ and $lfd24$. Similarly, we can do this for any two extended diagrams.

The following flowchart is the algorithm for constructing a diagram morphism between two extended diagrams.

Fig. 24



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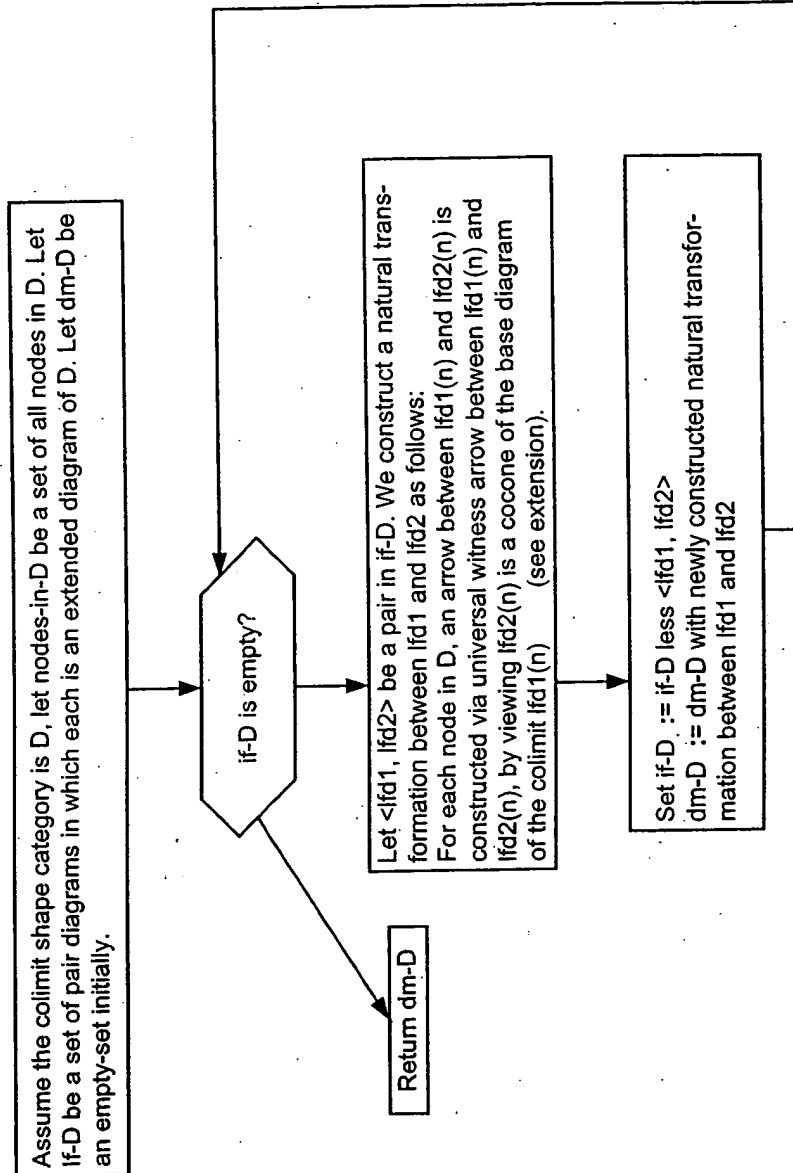
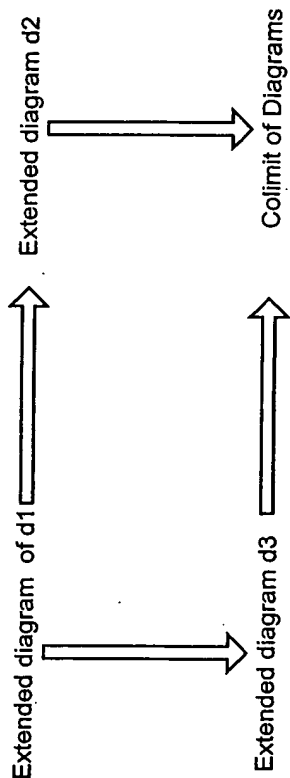


Fig. 25



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The final step is to complete the colimit of the extended diagrams. The colimit is determined by computing the pointwise colimits over corresponding nodes in the extended diagrams. The morphisms are computed uniquely using universality of the pointwise colimits.



Taking Pointwise Colimit of Extended Diagrams
(Can be done, since extended diagrams are all the same shape)

Fig. 26



Express Mail No. EV442675628 US
Application No. 09/665,179
Title: Method & Apparatus for Determining
Colimits of Hereditary Diagrams
Inventors: Dusko Pavlovic, et al.
Atty. Docket No.: 11128-04483

Diagram	Hereditary Diagram			Arc	Shape Morphism Graph	
Arc; source and target nodes	Arc; source and target diagrams	...	Arc; source and target diagrams	Shape functor (Fe)	Diagram Category Pair	
...	Natural Transformation (Fn)	Arc	
Arc; source and target nodes	Arc; source and target diagrams	...	Arc; source and target diagrams			
Total number of Arcs	Total number of Arcs		Total number of Arcs			

Examples of Data Structures used in the Example Implementation
Fig. 27



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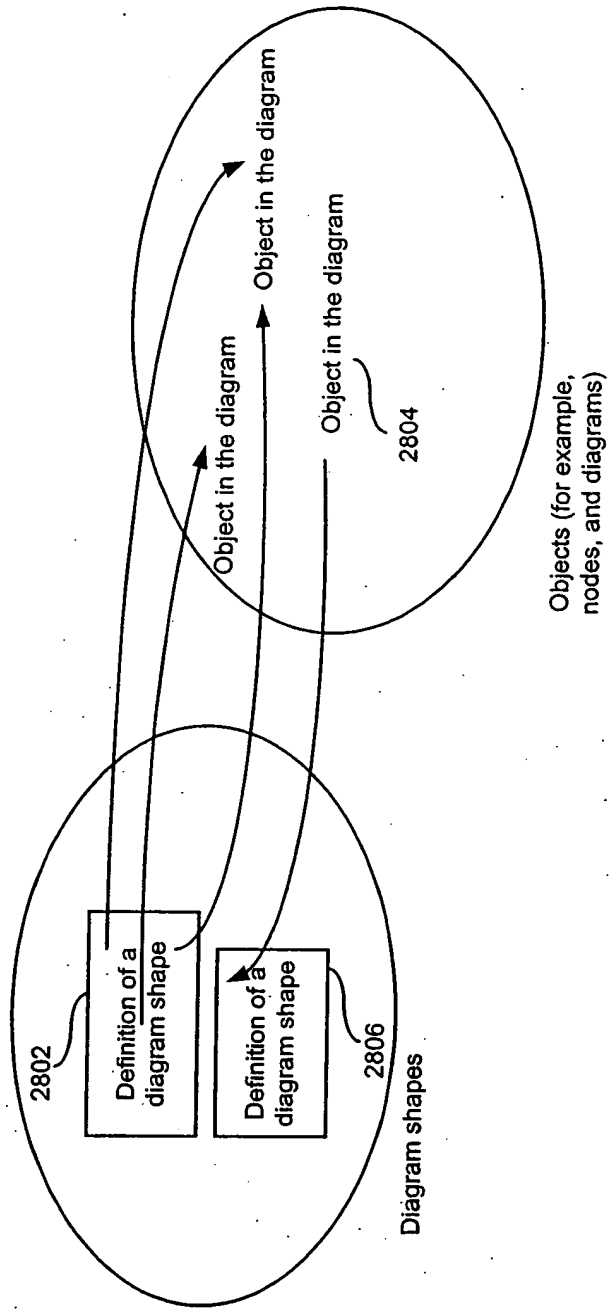


Fig. 28



Express Mail No. EV442675628 US
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Inventors: Dusko Pavlovic, et al.
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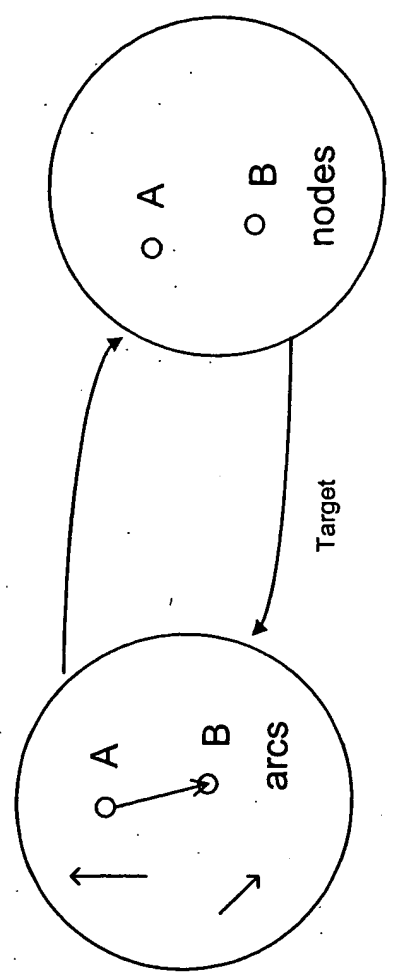


Fig. 29